Black holes as "gravitational Bohr's hydrogen atoms"

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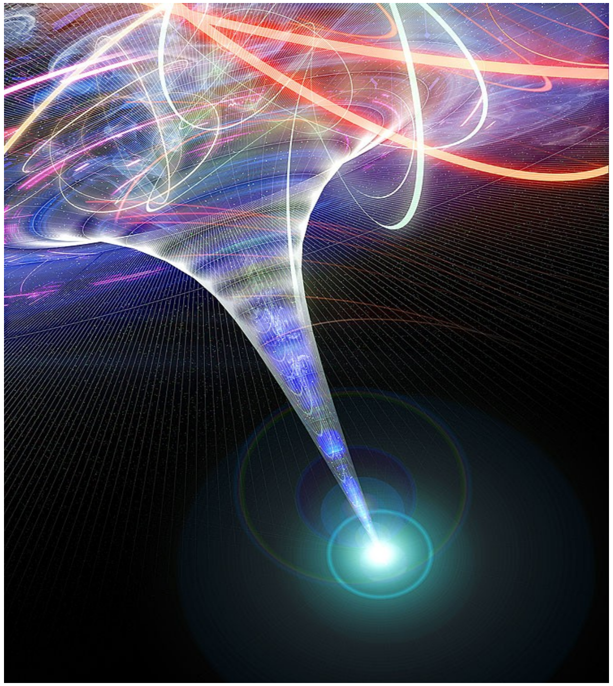
Black holes

The concept of black-hole (BH) has been considered very fascinating by scientists even before the introduction of general relativity (see C. Corda, *A clarification on the debate on "the original Schwarzschild solution"*, Electron. J. Theor. Phys. 8, 25, 65-82 (2011) for an historical review).

A BH is a region of space from which nothing, not even light, can escape. It is the result of the deformation of spacetime caused by a very compact mass. Around a BH there is an undetectable surface which marks the point of no return. This surface is called an event horizon. It is called "black" because it absorbs all the light that hits it, reflecting nothing, just like a perfect black body in thermodynamics.

The root for quantum gravity

BHs arise from the gravitational collapse of very massive stars, which is one of the greatest processes in Nature. However, an unsolved problem concerning such objects is the presence of a space-time singularity in their core. Such a problem was present starting from the first historical papers concerning BHs. It is a common opinion that this problem could be solved when a correct quantum gravity theory will be, finally, obtained, see Gen. Rel. Grav. 41, 4, 673-1011, *Special Issue on quantum gravity*, April 2009, for recent developments.

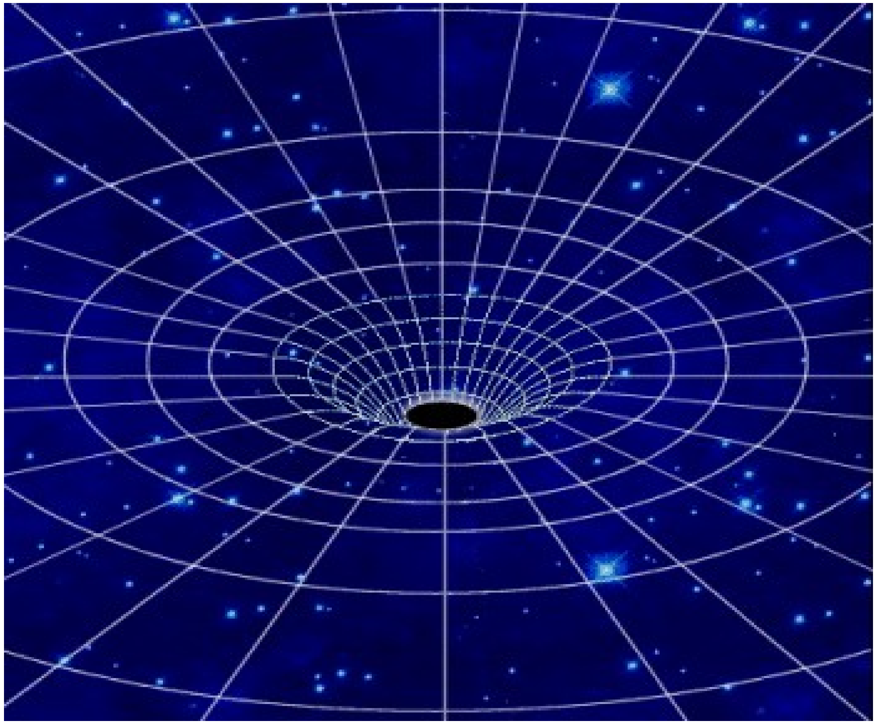


Thus, BHs are considered theoretical laboratories for testing models of a yet unknow unitary quantum gravity theory. We need a more general unitary theory, i.e. a quantum gravity theory, which will permit to unify general relativity and quantum mechanics in order to totally understand all the properties of BHs.

Entropy, information and black hole's evaporation

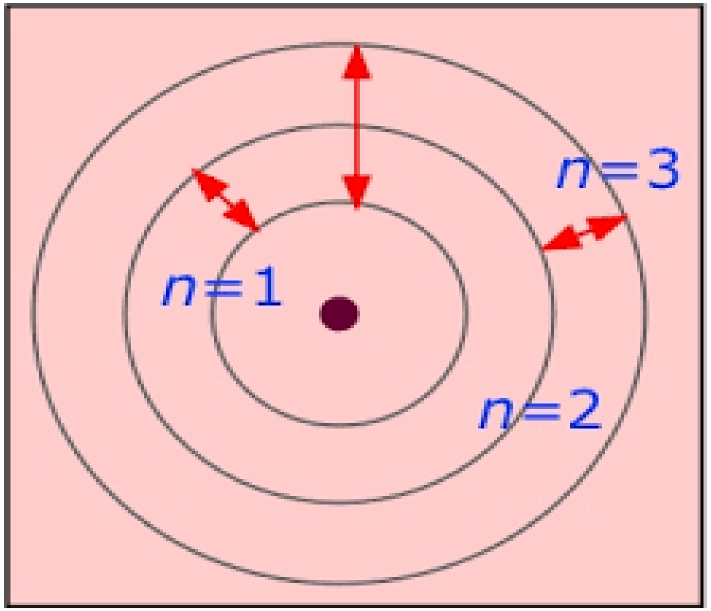
Hawking radiation (S. W. Hawking, Commun. Math. Phys. 43, 199 (1975)) and BH's entropy (J. D. Bekenstein, Nuovo Cim. Lett. 4, 737 (1972)) are the two most important predictions of a yet unknown unitary quantum theory of gravity. They are also connected to the famous BH information paradox (S. W. Hawking, Phys. Rev. D 14, 2460 (1976): will information be lost in the process of BH's evaporation?).

A remarkable result by Burinskii Gen. Rel. Grav. 41, 2281-2286 (2009) Firts Prize Winner at GRF 2009, showed that, for the Kerr-Schild BH "*interior of a BH is not isolated from the exterior region and matter may leave the interior in the form of the outgoing null radiation, and so, there is no information loss paradox*".



The “gravitational atom”

Researchers in quantum gravity have the intuitive, common conviction that, in some respects, BHs are the fundamental bricks of quantum gravity in the same way that atoms are the fundamental bricks of quantum mechanics (Bekenstein). This similarity suggests that the BH mass should have a discrete spectrum. On the other hand, the analogy generates an immediate and natural question: if the BH is the nucleus of the “gravitational atom” in quantum gravity, what are the electrons? In this lecture,we will give an intriguing answer to this question.



Non-thermal spectrum and effective temperature

Differently from Hawking's original computation, by using the *"tunnel approach"*, Parikh and Wilczek showed that the radiation spectrum cannot be strictly thermal (M. K. Parikh and F. Wilczek, Phys. Rev. Lett. 85, 5042 (2000)). Parikh released an intriguing physical interpretation of this fundamental issue by discussing the existence of a secret tunnel through the BH's horizon (M. K. Parikh, Gen. Rel. Grav. 36, 2419 (2004), First Award at GRF 2004). By implementing energy conservation, the BH contracts during the process of radiation. Thus, the horizon recedes from its original radius to a new, smaller radius. Hence, BHs cannot strictly emit like black bodies. This has profound implications for the BH information puzzle because arguments that information is lost during BH evaporation rely in part on the assumption of strict thermality of the spectrum.

In Hawking's original computation the probability of emission is (we use Planck units, G=c=k_B=1)

$$\Gamma \sim \exp(-\frac{\omega}{T_H})$$

$$T_H \equiv \frac{1}{8\pi M}$$ **is the Hawking temperature**

ω is the frequency of the emitted radiation.

The remarkable correction by Parikh and Wilczek, due by an exact calculation of the action for a tunnelling spherically symmetric particle, yields

$$\Gamma \sim \exp[-\frac{\omega}{T_H}(1-\frac{\omega}{2M})]$$

This result has also taken into account the conservation of energy and this enables a correction, the additional term $\omega/2M$. Let us introduce the effective temperature (which depends on the energy-frequency of the emitted radiation)

$$T_E(\omega) \equiv \frac{2M}{2M-\omega} T_H = \frac{1}{4\pi(2M-\omega)}$$

The probability of emission can be rewritten in Boltzmann-like form

$$\Gamma \sim \exp[-\beta_E(\omega)\omega] = \exp\left(-\frac{\omega}{T_E(\omega)}\right)$$

The effective temperature and the following analysis have been introduced in C. Corda, JHEP 08, 101 (2011) and refined in C. Corda, Int. Journ. Mod. Phys. D 21, 1242023 (2012, Honorable Mention in the Gravity Research Foundation Competition) in C. Corda, Eur. Phys. J. C 73, 2665 (2013) and in other papers.

$$\mathrm{Exp}[-\beta_E(\omega)\omega]$$

is the effective Boltzmann factor where

$$\beta_E(\omega) \equiv \frac{1}{T_E(\omega)}$$

The ratio

$$T_E(\omega)/T_H = 2M/(2M-\omega)$$

represents the deviation of the radiation spectrum of a BH from the strictly thermal feature. In other terms, as the correction by Parikh and Wilczek implies that a BH does not strictly emit like a black body, the effective temperature represents the temperature of a black body that would emit the same total amount of radiation.

Finalizing the Parikh-Wilczek analysis

I have recently finalized the Parikh and Wilczek tunnelling picture showing that the Parikh-Wilczek probability of emission is indeed associated to the two distributions

$$\langle n \rangle_{boson} = \frac{1}{\exp(8\pi M_E \omega) - 1} = \frac{1}{\exp[4\pi(2M - \omega)\omega] - 1}$$

$$\langle n \rangle_{fermion} = \frac{1}{\exp(8\pi M_E \omega) + 1} = \frac{1}{\exp[4\pi(2M - \omega)\omega] + 1},$$

for bosons and fermions respectively, which are non-strictly thermal, C. Corda, Ann. Phys. 337, 49 (2013).

Black hole's quasi-normal modes

The physical interpretation of BH's quasi-normal modes (QNMs) is fundamental for realizing unitary quantum gravity theory as BHs are considered theoretical laboratories for testing models of such an ultimate theory and their QNMs are natural candidates for an interpretation in terms of quantum levels. In thermal approximation and for large *n* (the principal quantum number), the frequencies of quasinormal modes for the Schwarzschild BH have the structure (L. Motl, Adv. Theor. Math. Phys. 6 (2003) 1135)

$$\omega_n = \ln 3 \times T_H + 2\pi i\left(n+\tfrac{1}{2}\right) \times T_H + \mathcal{O}\left(n^{-\frac{1}{2}}\right) =$$

$$= \frac{\ln 3}{8\pi M} + \frac{2\pi i}{8\pi M}\left(n+\tfrac{1}{2}\right) + \mathcal{O}\left(n^{-\frac{1}{2}}\right).$$

•

BH QNMs are frequencies of radial spin perturbations which obey a time independent Schröedinger-like equation (L. Motl, Adv. Theor. Math. Phys. 6 (2003) 1135). They are the BH modes of energy dissipation which frequency is allowed to be complex. They are the oscillation of the BH horizon due to external perturbations. By using *Bohr's Correspondence Principle*, which states that *"transition frequencies at large quantum numbers should equal classical oscillation frequencies",* Hod has shown that QNMs release information about the area quantization as QNMs are associated to absorption of particles (S. Hod, Gen. Rel. Grav. 31, 1639 (1999, Fifth Award in the Gravity Research Foundation Competition)). Hod's work was refined by Maggiore (M. Maggiore, Phys. Rev. Lett. 100, 141301 (2008)), who solved some important problems.

On the other hand, as QNMs are countable frequencies, ideas on the continuous character of Hawking radiation did not agree with attempts to interpret QNMs in terms of emitted quanta, preventing to associate QNMs modes to Hawking radiation (L. Motl, Adv. Theor. Math. Phys. 6 (2003) 1135).

Bohr-like model of black holes

Recently, Zhang, Cai, Zhan and You (B. Zhang, Q. Cai, M. Zhan, L. You, Int. J. Mod. Phys. D 22, 1341014 , First Award Gravity Research Foundation 2013) and myself and collaborators (C. Corda, JHEP 08, 101 (2011), C. Corda, Int. Journ. Mod. Phys. D 21, 1242023 (2012), C. Corda, S. H. Hendi, R. Katebi, N. O. Schmidt, JHEP 06, 008 (2013), C. Corda, Eur. Phys. J. C 73, 2665 (2013), etc. observed that the non-thermal spectrum by Parikh and Wilczek also implies the countable character of subsequent emissions of Hawking quanta. This issue enables a natural correspondence between QNMs and Hawking radiation, permitting to interpret QNMs also in terms of emitted energies. In fact, QNMs represent the BH reaction to small, discrete perturbations in terms of damped oscillations. The capture of a particle which causes an increase in the horizon area is a type of discrete perturbation. Then, it is very natural to assume that the emission of a particle which causes a decrease in the horizon area is also a perturbation which generates a reaction in terms of countable QNMs as it is a discrete instead of continuous process. On the other hand, the correspondence between emitted radiation and proper oscillation of the emitting body is a fundamental behavior of every radiation process in Science.

From the quantum mechanical point of view, one considers Dirac delta perturbations which represent subsequent absorptions of particles having negative energies which are associated to emissions of Hawking quanta in the mechanism of particle pair creation. Based on such a natural correspondence between Hawking radiation and BH QNMs, one can consider QNMs in terms of quantum levels also for emitted energies. This important point is in agreement with the general idea that BHs can be considered in terms of highly excited states in an underlying quantum gravity theory.

C. Corda, Class. Quant. Grav. 32, 195007 (2015).
C. Corda, Adv. High En. Phys. 867601 (2015).

To take into due account the deviation from the thermal spectrum, one must replace the Hawking temperature T_H with the effective temperature T_E in the equation of QNMs, see C. Corda, Eur. Phys. J. C 73, 2665 (2013), C. Corda, Adv. High En. Phys. 867601 (2015):

$$\omega_n = a + ib + 2\pi i n \times T_E(|\omega_n|)$$

$$\simeq 2\pi i n \times T_E(|\omega_n|) = \frac{in}{4M - 2|\omega_n|},$$

An intuitive explanation is the following. In thermal approximation, the QNMs determine the position of poles of a Green's function on the given background and the Euclidean BH solution converges to a thermal circle at infinity with the inverse temperature $1/T_H$ (see L. Motl and A. Neitzke, Adv. Theor. Math. Phys. 7 (2003) 307). Thus, the spacing of the poles in the QNMs equation coincides with a spacing which scales like T_H expected for a thermal Green's function.

But, if we want to consider the deviation from the thermal spectrum, it is natural to assume that the Euclidean BH solution converges to a non-thermal circle at infinity. Therefore, it is straightforward the replacement

$$\beta_H = \frac{1}{T_H} \rightarrow \beta_E(\omega) = \frac{1}{T_E(\omega)}$$

which takes into account the deviation of the radiation spectrum of a BH from the strictly thermal feature. In this way, the spacing of the poles coincides with a spacing which scales like $T_E(\omega)$ expected for a non-thermal Green's function (a dependence from the frequency is present).
A rigorous explanation is the following. QNMs are frequencies governed by a Schrodinger equation with the famous Regge-Wheeler potential. In Adv. Theor. Math. Phys. 6, 1135 (2003), Motl solved such Schrodinger equation satisfying purely outgoing boundary conditions both at the horizon
(r = 2M) and in the asymptotic region (r = infinity).

But, if we want to take into due account the conservation of energy, we have to replace the original BH's mass M in the Schrodinger equation with an effective mass of the contracting BH. In other words, if M is the initial mass of the BH before the emission, and $M - \omega$ is the final mass of the BH after the emission, the conservation of the energy enables the introduction of the effective mass

$$M_E \equiv M - \frac{\omega}{2}$$

of the BH during the emission of the particle, i.e. during the contraction's phase of the BH.
If one realizes step by step the same rigorous analytical calculation in by Motl, satisfying purely outgoing boundary conditions both at the effective horizon ($r = 2M_E$) and in the asymptotic region (r = infinity), the final result will be, obviously and rigorously, the equation of QNMs where T_E replaces T_H, C. Corda, Eur. Phys. J. C 73, 2665 (2013), C. Corda, Adv. High En. Phys. 867601 (2015).

Considering the leading asymptotic behavior the physical solution is

$$E_n \equiv |\omega_n| = M - \sqrt{M^2 - \frac{n}{2}}.$$

which is interpreted like the total energy emitted by the BH at that time, i.e. when the BH is excited at a level n.
Considering an emission from the ground state (i.e. a BH which is not excited) to a state with large $n=n_1$, the BH mass changes from M to

$$M_{n_1} \equiv M - E_{n_1} = \sqrt{M^2 - \frac{n_1}{2}}.$$

In the transition from the state with $n=n_1$ to a state with $n=n_2$ where $n_2 > n_1$, the BH mass changes again to

$$M_{n_2} \equiv M_{n_1} - \Delta E_{n_1 \to n_2} = M - E_{n_2} = \sqrt{M^2 - \frac{n_2}{2}},$$

where

$$\Delta E_{n_1 \to n_2} \equiv E_{n_2} - E_{n_1} = M_{n_1} - M_{n_2} = \sqrt{M^2 - \frac{n_1}{2}} - \sqrt{M^2 - \frac{n_2}{2}},$$

is the jump between the two levels due to the emission of a particle. Thus, in our BH model, during a quantum jump a discrete amount of energy is radiated and, for large values of the principal quantum number n, the analysis becomes independent from the other quantum numbers. In a certain sense, QNMs represent the "electron" which jumps from a level to another one and the absolute values of the QNMs frequencies represent the energy "shells". The model is in turn analogous to the historical semi-classical model of the structure of a hydrogen atom introduced by Bohr in 1913. In that model electrons can only gain and lose energy by jumping from one allowed energy shell to another, absorbing or emitting radiation with an energy difference of the levels according to the Planck relation. In our BH model, QNMs can only gain and lose energy by jumping from one allowed energy shell to another, absorbing or emitting radiation (emitted radiation is given by Hawking quanta) with an energy difference of the levels according to the above equation.

The similarity is completed if one notes that the interpretation is of a particle (the "electron") quantized on a circle of length

$$L = \frac{1}{T_E(E_n)} = 4\pi\left(M + \sqrt{M^2 - \frac{n}{2}}\right),$$

which is the analogous of the electron travelling in circular orbits around the hydrogen nucleus, similar in structure to the solar system, of Bohr model. On the other hand, Bohr model is an approximated model of the hydrogen atom with respect to the valence shell atom model of full quantum mechanics. In the same way, our BH model should be an approximated model with respect to the definitive, but at the present time unknown, BH model arising from a full quantum gravity theory.
There are various important consequences of the above approach on the quantum physics of BHs starting by the area quantization. In Bekenstein's original computation in Lett. Nuovo Cim. 11, 467 (1974) the area quantum of the Schwarzschild BH was

$$\Delta A = 8\pi$$

Setting $n-1=n_1$, $n=n_2$, we get the emitted energy for a jump among two neighboring levels as

$$\Delta E_{n-1\to n} = \sqrt{M^2 - \frac{n+1}{2}} - \sqrt{M^2 - \frac{n}{2}}.$$

The BH horizon area A is related to the mass through the relation $A=16M^2$. Thus, the variation of the mass is connected with the variation of the area by

$$\triangle A = 32\pi M \triangle M$$

Then, by using the effective mass corresponding to the transitions between the two levels n and n – 1, which is the same for emission and absorption, we get

$$|\triangle A_n| = |\triangle A_{n-1}| = 8\pi.$$

Thus, we retreive the famous Bekenstein's result on the area quantization

This analysis will have important consequences on BH entropy. Assuming that, for large n, the horizon area is quantized our approach gives the number of quanta of area before the emission as

$$N_{n-1} \equiv \frac{A_{n-1}}{|\triangle A_{n-1}|} = \frac{16\pi M_{n-1}^2}{32\pi M_{E(n,\, n-1)} \cdot \Delta E_{n-1\to n}} = \frac{M_{n-1}^2}{2M_{E(n,\, n-1)} \cdot \Delta E_{n-1\to n}}$$

and after the emission as

$$N_n \equiv \frac{A_n}{|\triangle A_n|} = \frac{16\pi M_n^2}{32\pi M_{E(n,\, n-1)} \cdot \Delta E_{n-1\to n}} = \frac{M_n^2}{2M_{E(n,\, n-1)} \cdot \Delta E_{n-1\to n}}$$

The famous formula of Bekenstein-Hawking entropy now becomes a function of the overtone number n. Before the emission

$$(S_{BH})_{n-1} \equiv \frac{A_{n-1}}{4} = 8\pi N_{n-1} M_{n-1} \cdot \Delta E_{n-1\to n} = 4\pi \left(M^2 - \frac{n+1}{2} \right)$$

After the emission

$$(S_{BH})_n \equiv \frac{A_n}{4} = 8\pi N_n M_n \cdot \Delta E_{n-1\to n} = 4\pi\left(M^2 - \frac{n}{2}\right)$$

In the limit *n* => infinity the emitted energy =>1/4M, and we re-obtain the standard result in literature (see for example A. Barvinsky, S. Das and G. Kunstatter, Class. Quant. Grav. 18 (2001) 4845)

$$S_{BH} \to 2\pi N.$$

In any case, it is a general belief that here is no reason to expect that Bekenstein-Hawking entropy will be the whole answer for a correct theory of quantum gravity. In order to have a better understanding of BH's entropy, it is imperative to go beyond Bekenstein-Hawking entropy and identify the sub-leading corrections. In Phys. Lett. B 668 (2008) 353, Zhang used the quantum tunnelling approach to obtain the sub-leading corrections to the second order approximation. In that approach, the BH's entropy contains three parts: the usual Bekenstein-Hawking entropy, the logarithmic term and the inverse area term

$$S_{total} = S_{BH} - \ln S_{BH} + \frac{3}{2A}.$$

The logarithmic and inverse area terms are the consequence of requesting to satisfying the unitary quantum gravity theory. A part from a coefficient, this correction to the BH's entropy is consistent with the one of loop quantum gravity. In fact, in loop quantum gravity the coefficient of the logarithmic term has been rigorously fixed at 1/2, see A. Ghosh and P. Mitra, Phys. Rev. D 71 (2005) 027502. By using our correction to Bekenstein-Hawking entropy we get

before the emission

$$(S_{total})_{n-1} = 4\pi\left(M^2 - \frac{n-1}{2}\right)$$

$$-\ln\left[4\pi\left(M^2 - \frac{n-1}{2}\right)\right] + \frac{3}{32\pi\left(M^2 - \frac{n-1}{2}\right)}$$

after the emission

$$(S_{total})_{n} = 4\pi\left(M^2 - \frac{n}{2}\right)$$

$$-\ln\left[4\pi\left(M^2 - \frac{n}{2}\right)\right] + \frac{3}{32\pi\left(M^2 - \frac{n}{2}\right)}$$

that in the limit n goes to infinity becomes

$$S_{total} \to 2\pi N - \ln 2\pi N + \frac{3}{16\pi N}.$$

Our results are in perfect agreement with previous literature.
Our Bohr-like model for BHs has important implications for the BH information paradox. In fact, this Lecture has shown that BH QNMs are really the BH quantum levels in our Bohr-like semi-classical approximation. This point implies that BHs are well defined quantum mechanical systems, having ordered, discrete quantum spectra, in perfect agreement with the unitarity of the underlying quantum gravity theory and with the idea that information should come out in BH evaporation. The time evolution of the model obeys indeed a Time-Dependent Schrodinger Equation and information comes out:
C. Corda, Ann. Phys., 353, 71, (2015).

Conclusion remarks

It is an intuitive but general conviction that BHs result in highly excited states representing both the "hydrogen atom" and the quasi-thermal emission in quantum gravity. In this Lecture we have shown that such an intuitive picture is more than a picture, discussing a model of quantum BH somewhat similar to the historical semi-classical model of the structure of a hydrogen atom introduced by Bohr in 1913. Our model has important implications for the BH information paradox and is in perfect agreement with existing results in the literature, starting from the famous result of Bekenstein on the area quantization.

Work in progress

1) It will be needed to generalize the analysis to all the types of BHs (the current analysis concerns the Schwarzschild BH).
2) It will be needed to quantize the model for the full strong field generated by the BH. In other word, the dream is to create a "QNMs quantum gravity".
4) Collaboration with Indian researchers of the Department of Mathematics, Jadavpur University, Kolkata West Bengal, India.
5) Collaboration with the Mahani Mathematical Research Center, Shahid Bahonar University of Kerman, Kerman, Iran for a connection between topological entropy and Bohr-like black holes.
6) Creation of a strong research group on black hole physics in the Research Institute for Astronomy and Astrophysics of Maragha involving students and researchers.

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